A Multidisciplinary Educational Path on the Concept of Infinity: The Traditional Way and the Contribution of AI

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Abstract

Aim. This paper deals with an educational path on the concept of infinity, a fundamental concept in mathematics and in every field of human theoretical and applied knowledge. The aspects of a traditional way are considered and compared to the possible contribution of AI.

Methods. A possible traditional multidisciplinary path is presented and compared with the results obtainable through the use of ChatGPT and MathGPT. It starts from the definition of this basic concept, passing through the difference between potential and actual infinity, the concept of infinitesimal, and some paradoxes of the infinity. The analysis is then extended to physics, philosophy and art.

Results. The traditional approach appears to be adequate and well-structured, with the possibility of interesting and fruitful additions; the approach through AI does not seem to add further positive elements, and brings with it inevitable underlying problems, relating to transparency, responsibility and privacy.

Conclusions. The non-technical but rigorous treatment allows the use of this work as an interesting educational introduction to one of the most important concepts that the human being has always dealt with. At the same time, it reveals the unavoidable limitations of AI.

Keywords: Mathematics, Infinity, Infinitesimal, Education, Knowledge, AI, ChatGPT, MathGPT.

Introduction: Historical Notes about the Concept of Infinity

Mathematics has always been an integral part of human culture and thought. The concept of infinity has been a particularly fascinating topic since man began

to think about himself, about the meaning of his life and about the world around him; it is at the basis of most mathematical thought, with strong connections to philosophy and spiritual disciplines (Di Sia, 2014, 2019; Zellini, 1993).

The following statement is traced back to the German mathematician David Hilbert (1862-1943): "The infinity! No other problem has ever shaken the human spirit so profoundly; no other idea has ever stimulated his intellect so profitably; and yet no other concept is in greater need of clarification than that of the infinity" (Bottazzini, 2017).

Infinity is a peculiar concept in mathematics; just to give a few examples, we speak of the "infinite extendability of a straight line", of the possibility of "dividing a segment into n equal parts", where n can be "as large as we want", of "approximating the area of a circle with regular inscribed and circumscribed polygons", etc. In mathematical analysis, which is also called "infinitesimal analysis", infinity is continuously present in every problem and is naturally used.

It is not possible to speak of fundamental concepts of mathematics such as real number, continuity, derivability and integrability of a function, without using the notion of limit of a function. This notion, which in the history of mathematics has required a long time for a rigorous definition, continuously uses the concept of infinity (Di Sia, 2017).

Mathematical analysis is the basis of all scientific studies, in particular it is the language of physics and of all scientific and technological disciplines that use mathematical tools (Di Sia, 2023; Ellis & Di Sia, 2023). Infinity has been and is the cause of paradoxes and antinomies/ contradictions; for example, when we deal with infinite sets, the statement "the whole is greater than the part" is no longer valid.

One of the greatest revolutions in mathematical thought took place in the second half of the nineteenth century when set theory was introduced, mainly by the German mathematician Georg Cantor (1845-1918); it would form the basis of all twentieth-century mathematics.

Cantor introduced the concept of "cardinality" of a set and demonstrated that the cardinality of the set of real numbers, also called "cardinality of the continuum", is greater than the cardinality of the set of natural numbers, called "cardinality of the countable". This means that it is impossible to put the set of natural numbers in one-to-one correspondence with the set of real numbers. Therefore he discovered, measured and classified the "transfinite" (Ewald, 2007; Lazzari, 2008).

Before the Greek period, mathematics was mainly linked to practical everyday problems; prior to this time, no particular information was found relating to reflections on abstraction and on the concept of infinity. With the Greeks, mathematics began to acquire the characteristics of a true science. The Greek taste for proportions and harmony transcended purely practical questions, giving mathematics an abstract dimension. The finite natural numbers were the key for measuring nature and establishing its relationships and the laws known at the time.

Regarding the concept of infinity, we briefly indicate the reflections of authoritative ancient thinkers:

A) According to the Greeks, the concept of infinity (apeiron) is linked to that of indefinite, indeterminate, imperfect. Therefore, the concept of infinity had a negative connotation for them.

B) With Pythagoras the natural number was the basis of everything; this opinion was then shared by authoritative figures many centuries later. Real concepts and situations were connected to each natural number, such as:
   a) the mind;
   b) the opinion (that breaks the unity);
   c) the completeness (the beginning, the middle, the end);
   d) the justice (the square, the exact).

C) For Plato the Good is (de)finite.

D) Aristotle makes a subtle distinction between potential and actual infinity, asserting that "the infinity has a potential existence, an actual infinity does not exist". The set of natural numbers is a potential infinity (because numbers continue to indefinitely grow, without a maximum value), but not actual infinity; it is possible to follow the sequence indefinitely, but not grasp its totality.

E) Zeno of Elea, in the 4th century BC, proposed the famous "paradox of Achilles and the turtle", which in fact involves the concepts of infinitesimal, infinity and infinite divisibility. Achilles accepts a running race with a turtle, which starts with a finite spatial margin of advantage; for every metre Achilles travels, the turtle travels 1 centimeter. Therefore, every time he reaches the position occupied by the turtle, it has advanced a little, although if less and less; the path that Achilles must take is in fact infinitely divisible (Berti, 1997; Di Sia, 2016).

   From a practical point of view, Achilles reaches and surpasses the turtle, but from a logical-mathematical point of view we fall into a possible paradox, if we consider that the space traveled each time by the turtle can be infinitely divided and that it will always be in front of Achilles, even if by a progressively smaller space, when he has reached the position of the turtle.

   From a mathematical point of view, it involves the calculation of a sum of infinite terms of the type:

$$1/2 + 1/4 + 1/8 + 1/16 + \ldots$$

   This sum is calculable, even if made up of an infinite number of addends, and admits a finite result.

F) Euclid studied the concept of infinity in a deep way, also giving an elegant demonstration of the fact that prime numbers are infinite.

G) Archimedes was interested in the number π, the ratio between the length of a circumference and that of its diameter. He developed innovative techniques for its calculation, in particular that of approximating the length of the circumference:
   a) by defect, through the perimeters of regular polygons inscribed in it;
   b) by excess, through the perimeters of regular polygons circumscribed to it.

He noticed that as the number n of sides of the polygons increased, the measurement became more and more correct, closer and closer to the true value, with n tending to infinite. He made estimates for finite, but increasing n. His method prefigures an infinite approximation process. The fact that numbers like π or $\sqrt{2}$ cannot be expressed as a fraction was still unknown to the Greeks and Archimedes.

Defining the Infinity: Potential and Actual Infinity

The symbol "∞", which symbolically denotes the mathematical concept of infinity, was introduced by John Wallis (1616-1703), an English mathematician, who contributed to the development of infinitesimal calculus (Maierù, 2007).

With infinities, their inverse realities also appeared in mathematics, i.e. "infinitesimals".

A) Isaac Newton (1642-1726) and Gottfried Wilhelm von Leibniz (1646-1716), initiators of the differential calculus, fundamental in providing an adequate mathematical foundation for the physical study of variable quantities such as velocity and acceleration, used:
   a) the symbol "∞" for the potential infinity;
   b) the symbol "dt" for the infinitesimal, or, according to Newton's expression, the "fluxion" (Di Sia, 2017).

   These innovations were initially contested by mathematicians of the time.

B) George Berkeley (1685-1753), Irish philosopher, theologian and bishop, one of the three great British empiricists together with John Locke and David Hume, in the work "The analyst: or a discourse addressed to an infidel Mathematician", defined the infinitesimals as "ghosts of evanescent entities" (Marciano, 2011).

C) Jean-Baptiste Le Rond d'Alembert (1717-1783), French encyclopedist, mathematician, physicist, philosopher and astronomer, among the most important protagonists of the "Enlightenment", proved to be more optimistic, stating: "Go ahead: faith will come to you" (D'Alembert & Diderot, 1966).

   In fact, already in the Middle Ages it was possible to calculate sums of infinite terms:

D) Nicholas of Oresme (1323-1382), French mathematician, physicist, astronomer and economist, bishop, philosopher, psychologist and musicologist, had demonstrated that some sums of numbers were infinite and understood that "infinitely divisible" does not imply "infinite" (Grant, 1974; Conway & Guy, 1999).

   Once again it is worth considering what the ancient Greeks thought about this; mathematics was born long before them, but neither the Egyptians, nor the Babylonians and the Chinese were interested in dealing with the infinity, preferring questions much closer to everyday life.

   Euclid had actually demonstrated that prime numbers are infinite; therefore, for understanding what happened in practice, it is advisable to proceed with a

careful reading of the texts. He does not state that a straight line is infinite, but that each segment of the straight line can be extended at will; nor that the prime numbers are infinite, but that they are greater than any definite quantity.

The underlying concept was therefore the "use of finite quantities", with the idea of being able to "always have more, when needed". This mode of approach was not limited only to mathematics. In fact, looking at two of the greatest classical philosophers:

E) Plato stated that the actual infinity (the "hyperuranion") is in itself unknowable and humans must be satisfied with a vision of the "shadows" produced by it;

F) Aristotle theorised the potential infinity, that we can define as something that is "beyond what we can reach", but "in that direction", like the extension of the segment for Euclid.

The concept of "limit" was completely unknown to the Greeks. Working only on finite quantities made things much easier; the price to pay, however, was a heavy limitation on provable theorems.

The case of measuring the area of a circle, mentioned above, is famous. A succession of polygons inscribed in the circle is considered, so the difference between the area of the circle and their area is increasingly reduced; we consider then a succession of polygons circumscribed to the circle, so the difference between the area of the circle and their area is increasingly reduced.

At this point we can give upper and lower bounds to the value of the area of the circle; Archimedes was able to calculate that the value of π is between "3 + 10/71" and "3 + 1/7". At least until the Enlightenment (about 1700), almost all mathematicians claimed that "their methods were equivalent to that of exhaustion"; it was a good way to avoid possible criticism.

Infinitesimals and Infinites

From the Archimedean period, for more than a millennium, mathematics in the West did not make great progress; in India and Arabia new discoveries were made, but still linked to the finite. Driven by the first results that went beyond classical knowledge, such as the solution by radicals of third and fourth degree equations, mathematicians began to look for new (more or less empirical) methods to derive new formulas.

Infinitesimals are linked to the concept of infinite; if we think to divide a segment, while the length of its parts is reduced, their number increases; we tend to have "an infinite number of parts of zero length". The idea of adding an infinite number of entities of zero dimension (the parts of the segment in fact become points) became a very delicate issue; however, the idea of infinitesimal became more and more interesting among mathematicians of the time.

In the same period, what would later be called "infinitesimal calculus" was being born. Fermat had been one of the first ones, trying to calculate the area under a curve, seeing it as formed by a series of small rectangles (Fig. 1).

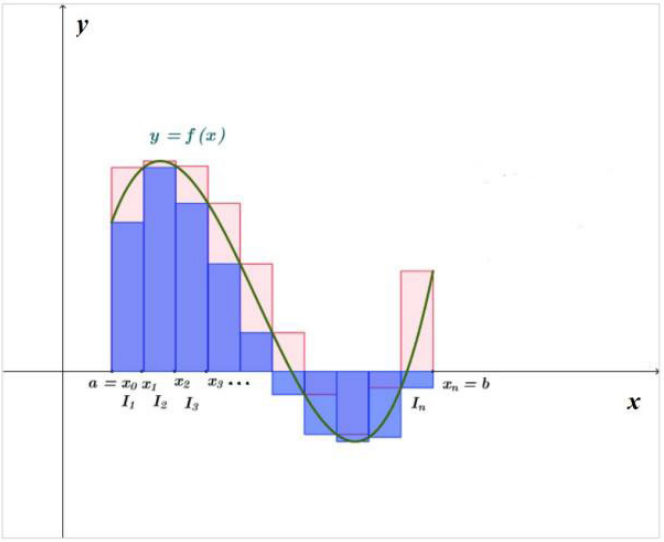

Fig. 1. Curvilinear area (trapezoid) as the sum of inscribed and circumscribed rectangles.
Source: Own arrangement.

Leibniz and Newton had laid the foundations for the practical use of derivation and integration formulas, among the complaints of many mathematicians. The other major trend where the infinity was consistently used has been that of series developments. In this field one of the undisputed fathers was Euler (1707-1783), a great formal mathematician, who studied the infinite series, not particularly worrying about the legitimacy of the steps as long as the final results determined by him made sense. He determined relations of the type:

$$1/(1 - x) = 1 + x + x^2 + x^3 + x^4 + \ldots..$$

By substituting numerical values for x, it was possible to "prove" that a number derived from a particular "infinite" sum of other numbers or fractions.

On the Paradoxes of Infinity

All these new results were interesting and important for the development of mathematics. There were, however, those who cautioned against generalisations without adequate proofs.

In relation to the concept of infinitesimal, in particular the ratio between two infinitesimals, these are two quantities that tend to zero, so the ratio seems to have an "undefined" meaning. If they are not zero, the operation of eliminating higher order infinitesimals did not seem legitimate.

Even with successions, there were problems. Taking for example the following sequence S of numbers, which at first sight seems to cause no problems:

$$S = 1 - 1 + 1 - 1 + 1 - \ldots..$$

It can be written as:

$$S = 1 - (1 - 1 + 1 - 1 + \ldots..),$$

that is $1 - S$; this implies the equality:

$$S = 1 - S,$$

which leads to the result $S = 1/2$, a result that does not seem reasonable, considering that the partial sums obtained by the starting sequence are always and only 1 and 0. Let us now suppose to rearrange the terms of the sequence, considering the two sequences that can be obtained from the starting one, one with the odd elements and one with the even ones. We have:

$$S_{odd} = -1 - 1 - 1 - 1 - 1 - \ldots..$$

$$S_{even} = 1 + 1 + 1 + 1 + 1 + \ldots..$$

Then S_{even} tends to $+\infty$, while S_{odd} tends to $-\infty$. In the event that one were to object that even and odd terms cannot be taken separately, it is necessary to explain why the commutative property holds for a finite sum and not for an infinite one. Another problem concerns why for normal quantities a quantity is always greater than its proper part, while with infinity this is not true.

In the 19th century, following a whole series of examples and counter-examples, a group of mathematicians managed to give a series of acceptable definitions for dealing with this type of problems related to infinity.

For infinitesimals, the fundamental step was the definition of the concept of limit. Instead of a division by zero, the key idea was to pay attention not to what happens at the point where we need to calculate the limit, but to look at what happens in the neighborhood of that point. Even the impossibility of defining a real number without its representation as a fraction had a negative impact at the beginning.

Regarding the possible convergence of the series, there were problems in relation to the possible presence of negative terms; to overcome this obstacle, it was decided to define an associated sequence, having as terms the absolute values of the original one. If this second series converges to a finite value, the original one is said "absolutely convergent" and the convergence problem is solved. Today's paradoxes are different from those of the ancient Greeks, but no less difficult to evaluate.

A further example is the following: every set that can be put in one-to-one correspondence with the set of natural numbers has the same cardinality as the natural numbers. The set of even numbers, which is a subset of the natural numbers, has the same cardinality as the set of natural numbers. In this sense, therefore, the part can be "equal" to the whole.

Infinity and Transfinity

In a few years, the treatment of the infinity had a notable evolution. The credit goes to Georg Cantor (1845-1918), German mathematician, father of modern set theory. He had the great merit of expanding the set theory to include within it the concepts of transfinite numbers, cardinal and ordinal numbers. He totally re-evaluated the point of view that had been maintained for more than two thousand years, relating to the actual infinity. He pointed out that numbers (finite and not finite) could be seen in two different ways:

a) "ordinal numbers", when we are interested in counting them;

b) "cardinal numbers", when we see them as a single group and we are interested in knowing "how many they are".

Having made this distinction, the next step was to work with ordinal infinite numbers seen as examples of actual infinity. To avoid talking about "infinite", he defined these numbers as "transfinite". Transfinite numbers extend the notion of number, the arithmetic operations and the order relationship typical of natural numbers to a wider class of objects, in some sense "bigger" than the usual "finite" numbers. As with finite numbers, there are two ways in which the notion of number can be extended to transfinite numbers, namely as ordinal numbers and as cardinal numbers.

Cantor began to suppose that the smallest transfinite ordinal, the one obtained by gradually counting all the integers, was a real entity, and he called it ω. The immediately following ordinal is $\omega + 1$, followed by $\omega + 2$, $\omega + 3$, etc., with the restriction that the commutative property of addition does not hold for such numbers: "$1 + \omega$" is not equal to "$\omega + 1$".

Regarding cardinal numbers, considering that we are interested in knowing how many they are, we must find a system to compare two cardinals, which is simple using finite numbers, not simple when they are infinite. Cantor introduced the following definition: "Two sets A and B have the same cardinality when their respective elements can all be associated in pairs with each other".

If, however, the correspondence is only in one sense, i.e. we can associate each element of A with an element of B, but some of the latter remain alone, then the cardinality of B, indicated as "card (B)", is greater or equal to that of A.

Regarding the name to be given to the transfinite cardinal numbers, Cantor used the first letter of the Hebrew alphabet, i.e. Alef (א), indicating the cardinality of the set of natural numbers with $\aleph_0$, Alef-zero. The next cardinal number is Aleph-one, $\aleph_1$. All sets of cardinality $\aleph_0$, called the "cardinality of the countable", are called countable, because we can count their elements with integers. Every set X equipotent to $\mathbb{N}$ (it is written $X \sim \mathbb{N}$) is called a "countable set".

Furthermore, unlike ordinal numbers, the commutativity of the sum applies to cardinals; put one pile first and then another, or vice versa, give the "same quantity" at the end.

Cantor proved that integers and rationals are sets of cardinality $\aleph_0$ and that real numbers are certainly more than $\aleph_0$, through a proof by contradiction: card $(\mathbb{R}) \neq \aleph_0$.

He posed by definition: card ($\mathbb{R}$) = $\mathfrak{C}$, that is, the cardinality of the continuum is $\mathfrak{C}$ (Gothic C). Since $\aleph_0 < \mathfrak{C}$, the cardinality of the countable is less than the cardinality of the continuum. $\aleph_0$ and $\mathfrak{C}$ are transfinite numbers; developing the theory, he found that infinite transfinite numbers exist.

Cantor also asked himself the following question: "There exists a set X such that $\mathbb{N} \subset X \subset \mathbb{R}$, so that we have: $\aleph_0 <$ card (X) $< \mathfrak{C}$?". In 1883 he answered in a negative way, but failed to prove it. This question remained open and went down in the history of mathematics with the name of "continuum hypothesis". Hilbert, in 1900, announced it as one of the mathematical open problems at the threshold of the 20th century.

Around 1930, Kurt Gödel (1906-1978), Austrian mathematician, logician and philosopher, known above all for his works on the incompleteness of mathematical theories and considered one of the greatest logicians of all time together with Aristotle and Frege, was able to demonstrate that the continuum hypothesis is "compatible" with the other axioms underlying the arithmetic we use, that is, if we want to accept it we do not fall into contradiction (Borga & Palladino, 1997; van Heijenoort, 1967; Shanker, 1991).

In 1963 the American mathematician Paul Cohen demonstrated, using an extremely ingenious method, that the continuum hypothesis is independent of the Zermelo-Fraenkel axioms on which the modern set theory is based (Jech, 2003). The word "independent" he used, means that in the Zermelo-Fraenkel system the continuum hypothesis can be neither proven nor refuted (Tourlakis, 2003).

The solution to the problem is therefore that it cannot be solved with the notions of set theory that are in common use today. This does not mean that extensions of set theory cannot emerge in the future that allow to decide on the continuum hypothesis in one sense or the other, but it only means that, for now, we have to settle separating the proven results using said hypothesis (or its denial) by those who instead do without it (Cohen, 2012; Di Sia, 2024).

Structure of a Possible Transdisciplinary Teaching Project on the Concept of Infinity

This paragraph outlines a possible multidisciplinary path on the concept of infinity through mathematics, physics, philosophy and art. The considered topics, both from a mathematical point of view and from that of historical-philosophical analysis, are part of the programmes provided by the Italian National Plan for Computer Science (*Il piano nazionale per l'informatica*).

The programme is designed as a laboratory activity, in which final-year high school classes engage in an exchange of experiences with their peers on the topic of mathematics, understood in an inter- and transdisciplinary sense. The teaching project can be integrated into the school's curriculum and can be completed entirely during school hours. The covered topics, not only from a mathematical perspective but also in relation to related disciplines, are

included in the programmes established by the National Plan for the respective academic years.

The project will involve all students in the class for approximately 5-6 months, during the middle of the school year, with 2-3 hours of dedicated time per week, which can be increased in the final phase.

The project, which spans multiple disciplines, aims to develop content and skills that will facilitate students' rational and rigorous approach to the concept of infinity, rather than merely intuitively. The two systems of thought, mathematical and philosophical ones, seek a rigorous framework for a concept that has always fascinated and tormented humanity.

Through the observation and description of infinity, the project aims to move from an incommensurability towards a gradual process of knowledge based on comparison, encouraging students to define the concept of infinity without resorting to purely negative or qualitative meanings.

The project is structured across multiple disciplinary areas and is aimed at acquiring content and skills that facilitate students in approaching the concept of infinity in rational and not just purely intuitive terms, allowing students to acquire cognitive tools useful for observing and describing the infinity.

From a mathematical point of view, the theme of infinity can be treated starting from the cardinality of sets; infinite numerical sets provide a very fruitful point of view to address the problems connected with infinity and to enhance the historicisation activity of mathematical thought.

The starting point is the discovery that infinite sets can be put in one-to-one correspondence with a proper subset of them and that, indeed, it is precisely this property that provides a valid criterion for distinguishing finite sets from infinite ones. The next step consists in demonstrating that N, Z and Q are equipotent and therefore have the same cardinality, defined as power of the countable.

The Cantor's discovery of the power of continuum and of an infinity greater than that of the countable is then considered, and by going through Cohen's conclusions, it is possible to trace a situation analogous to that created in geometry, with the birth and development of non-Euclidean geometries.

Students should be guided to discover the different problems with extreme graduality, through the constant use of heuristic procedures and examples, and with constant reference to the historical contextualisation of problems.

A) Regarding the "objectives", the development of the project aims to achieve the following educational and training purposes:

"Knowledge"

a) acquisition by students of mathematical tools that allow them to approach the concept of infinity in rational and not only intuitive terms, through the formulation of hypotheses and guided conjectures;
b) acquisition of a rigorous terminology, appropriate to different contexts;
c) knowledge of the historical-philosophical context in which the different problems have developed.

"Know-how"

d) development of the ability to use the acquired knowledge in solving problems related to the theory of finite and infinite sets;
e) development of the ability to analyse and develop simple interdisciplinary paths related to the covered topics;
f) development of the ability to create paper and digital dissemination materials with a view to transmitting knowledge.

"Knowing how to be"

g) development of the ability to work in a group through the awareness of one's own abilities and limits in relation to a pre-established objective;
h) maturation of the processes of socialisation and interaction related to a critical and conscious knowledge of one's own attitudes, and also of one's insecurities;
i) maturation of the critical capacity with respect to the communicative effectiveness in the transmission of knowledge, differentiating the proposal according to the age and knowledge of the listeners.

B) Regarding "content and analysis", students are encouraged to ask themselves mathematical and philosophical questions about the covered topics, such as:
- What does it mean that a set is infinite?
- Is it possible to define the concept of infinity without appealing to a negative connotation?
- Are natural numbers more than even (or odd) numbers?
- Is it possible to think of different types of infinity?
- Do infinite sets have the same properties as finite sets?
- Does thinking about multiple mathematical infinities authorise us to hypothesise infinite universes?
- Is physical infinity easier to conceive and accept than mathematical infinity?
- What does "infinite space" and "infinite time" mean?
- Did the universe have a beginning and will it have an end?
- Is it possible to visualise infinity in artistic terms, for example in terms of infinite cycles, subdivisions of segments, areas, volumes?

It is possible to study mathematics in different ways; the analysis of the historical evolution of concepts makes more comprehensible and easier the study of definitions and theorems. This leads to a statistical improvement in the students' profit levels. The need to explain to others the contents learned in a non-traditional way allows a greater internalisation of the proposed themes.

On the Involvement of ChatGPT and MathGPT

AI is taking on an important role in people's lives, as it aims to provide a whole series of completely new skills and ways of living within the social context.

In recent years, AI has had a growing development in the educational field, both at a theoretical and applied level, as it is able to transform the teaching method, as well as to modify learning, making it more compliant with the specific needs of each student. This is, at least, the theoretical aim.

AI also involves issues regarding the ethical and responsible use of technological supports, in terms of privacy of sensitive data concerning all involved parties. A fundamental assumption is that AI should always be seen "for its instrumental character that acts as a support to the people" who use it, and never as "an alternative to human interaction".

MathGPT surpasses ChatGPT in mathematical performance, even surpassing pre-existing models from competitors like OpenAI and Microsoft. This has significant implications for the education sector, making it an attractive tool for teaching and learning, considering that ChatGPT struggles with accuracy in specialised domains.

Unlike ChatGPT, which is trained using a wide variety of text data, MathGPT specialises in domain-specific training data; this gives it an advantage in mathematics as a particular area.

The features it offers can be summarised as follows:

a) Personalised maths assistance: it acts as a maths tutor for various aspects of mathematics learning.
b) Question generator: it generates similar maths questions at different difficulty levels, based on an initial prompt.
c) History of maths: it provides explanations associated with specific mathematical concepts.
d) Translation of maths questions: it provides translations for maths questions, facilitating comprehension in different languages.
e) Teaching support for different levels: it starts with primary school maths, adapting to different educational levels.
f) Analysis of random and logical phenomena: it helps to understand and solve problems related to random and logical phenomena in mathematics.
g) Calculation and counting modules: it provides tools for performing complex mathematical calculations and understanding the principles of counting.
h) Discussion and exploration of problems: it allows for the exploration and discussion of mathematical problems, encouraging an analytical approach.
i) Symbol substitution and imagination space: it helps in solving problems that require symbol manipulation.
j) Measurement modules and number theory: it provides support for measurement and number theory concepts.
k) Number operations and numerical understanding: it helps students in understanding number operations and numerical concepts.

l) Data analysis and organisation: it provides assistance in interpreting and organising statistical data.
m) Practical applications: it facilitates the application of mathematical concepts in practical situations.
n) Formulas memorisation: it supports memorisation of mathematical formulas.
o) Applied and common problem solving: it helps in solving everyday applied problems and common mathematical questions.
p) Combination and elimination: it provides methods for combining and eliminating elements in mathematical problems.

ChatGPT is a language model, trained to be a good conversationalist, but poorly trained to abstract and elaborate mathematical concepts. Recently, MathGPT, a language model developed by Mathpresso in collaboration with Qanda and Upstage, supported by KT, has attracted considerable interest.

MathGPT's superiority in mathematics appears to have significant implications for the education sector. Its ability to solve even complex mathematical problems makes it a valuable tool for teaching and learning, addressing the challenges presented by models like ChatGPT, which often struggle with accuracy in specialised areas. Unlike ChatGPT, which is trained on a wide range of text data, MathGPT has specialised in domain-specific training data. This appears to give it an advantage in mathematics, as well as in other specialty areas.

The offered functions can be summarised in these terms: personalised mathematical assistance, question generator, history of mathematics, translator of mathematical questions, teaching support for various levels, analysis of random and logical phenomena, calculation and counting modules, discussion and exploration of problems, space of imagination, measurement and number theory modules, operations on numbers and numerical understanding, analysis and organization of data, practical application and transition to higher levels of education, innovative thinking and memorisation of formulas, solving applied and common problems, logic and contradiction.

Regarding the practical use of MathGPT, before accessing (depending on the various online versions) it requires users to indicate the telephone number with the prefix to receive the access code via SMS. It is then possible to write any mathematical problem; the solution is returned, with a step-by-step explanation of each detail. It then offers the possibility of continuing to talk in the same chat or to create a new chat to request the resolution of a different problem (Adiguzel, Kaya, & Cansu, 2023; Göçen & Aydemirb, 2020; Cope & Kalantzis, 2019).

A Multidisciplinary Educational Path on the Concept of Infinity through MathGPT

When asked to create a multidisciplinary educational path on the concept of infinity, MathGPT (*MathGPT - AI Math Solver*) responded in these terms:

Certainly! Here's a multidisciplinary educational path on the concept of **infinity**, integrating mathematics, philosophy, science, art, and literature.

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1. Mathematics

- **Basic Notion:** Infinity (∞) represents an unbounded quantity; greater than any real number.
- **Types:**
  - **Potential Infinity:** Process that never ends (e.g., counting numbers).
  - **Actual Infinity:** Completed infinite totality (e.g., the set of all real numbers).
- **Set Theory:** Georg Cantor's work—different sizes of infinity (countable vs uncountable).
- **Calculus:** Limits approaching infinity, infinite series, and convergence/divergence.

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2. Philosophy

- **Ancient Views:** Zeno's paradoxes (can motion exist with infinite divisibility?).
- **Modern Philosophy:** Kant, Aristotle—debating actual vs potential infinity.
- **Metaphysical Questions:** Can infinity exist in reality, or only conceptually?

3. Science

- **Physics:** Is the universe infinite? Space and time concepts in cosmology and relativity.
- **Mathematical Physics:** Use of infinity in equations (e.g., black holes, singularities).
- **Biology:** Infinite patterns (fractals in nature).

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4. Art

- **Visual Art:** M.C. Escher's artwork exploring infinite patterns and recursion.
- **Music:** Infinite canons and musical sequences.

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5. Literature

- **Poetry/Prose:** Infinite themes in Jorge Luis Borges' stories ("The Library of Babel").
- **Symbolism:** Infinity as a metaphor for love, existence, or knowledge.

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6. Religion & Culture

- **Symbolic Meanings:** Infinity as a spiritual or eternal concept in various religions.

- **Cultural Symbols:** The infinity symbol (∞) representing endlessness.

Summary Table

Discipline	Key Idea of Infinity
Mathematics	Different sizes of infinity, infinite sets/series
Philosophy	Actual vs potential infinity; paradoxes
Science	Infinite universe, infinity in physical theories
Art	Infinite patterns and recursion
Literature	Infinity in stories, metaphors, and poetry
Religion/Culture	Infinity as eternal, unending, spiritual

Infinity is a profound concept that connects many fields, offering endless opportunities for exploration and understanding.

From a comparison between what was introduced before (and extendable in quantity and quality) and what is derived from MathGPT, we note that the latter is interesting and full of ideas, but offers a compact overview of what is possible to do through a traditional method. It is possible to ask for further help from MathGPT, but this approach does not seem in general to provide more information than an adequate, in-depth and duly structured traditional approach.

Challenges and Concerns of AI from a Pedagogical Point of View

An alarming aspect at a social level regarding the use of digital platforms that make use of AI concerns the concept of privacy and security of personal data. Systems that use AI have to do with the collection and analysis of large amounts of personal data.

What should be fundamental, also according to European regulations, concerns protection through security software that implies data encryption and access only to authorised personnel. This also results in a responsible transmission and use by people regarding what and how much they enter into these databases, for avoiding possible risks of data leakage, which could expose people's safety to danger and damage (Bostrom & Yudkowsky, 2014; *Report on artificial intelligence in education, culture and the audiovisual sector*, 2021).

With the development of these new technologies, which are profoundly influencing people's lives, especially those of young people (thinking also to

social media), a double effect occurs: on the one hand the advantage of a global connection with the rest of the world, on the other the disadvantage of isolating and distancing from reality, and from building healthy human relationships as happened before the advent of mobile devices.

As AI becomes more and more pervasive in the daily lives of every human being, this can lead people being exposed to risks without being fully aware of it. From the moment we come into possession of people's personal information, data ethics means that the requirement of transparency must be met.

Regarding ethical guidelines, AI systems should conform to current social values. Therefore, it is essential to have safety and human protection measures that monitor the pervasive modes of AI systems (Fagan & Levmore, 2019).

The management of collected data, with reference to the place and time of storage and especially to those who are granted the right to access and use information, remains a worrying aspect from an ethical point of view and should be protected with clear and well-defined regulations, something that has not yet been achieved to date (Hsu & Ching, 2023a, 2023b; Talan, 2021; Zhou, 2023).

About the ethical use of ChatGPT in the school, AI is a powerful tool, but it must be appropriately used; this must be made clear to teachers and taught to students. Studying is and must remain a moment of growth; it must not be bypassed by cheating with AI. When we talk about ethics in the school, we must not only refer to the necessary respect for privacy rules, but also to honesty in its use.

Chatgpt is to be considered a support, not a replacement. To emphasise the importance of a responsible AI use, teachers can follow simple but clear guidelines:

- establish clear rules, transparently specifying when and how to use it;
- conduct group exercises in the class to develop an understand the potential and challenges behind the use of AI;
- explain the importance of always verifying information;
- remember that, even if it is Chatgpt, the source must be always cited to avoid the risk of plagiarism.

It is essential to educate students about the proper use of AI in the school; clear ethical principles are needed to properly integrate it into daily activities, including studying.

Conclusions

The concept of infinity is one of the key concepts of all mathematical thought; it has aroused interest and fascination since the dawn of humanity, when man began to reason about the meaning of what surrounds him and his own existence. It is a concept present in all cultural, scientific and technological areas of reality, intimately connected to that of infinitesimal and nothingness (Di Sia, 2011, 2013, 2014).

It has led to various contradictions and paradoxes, and at the same time has helped in an ever fuller understanding of the reality that surrounds us and of which we are a part of. This concept helps well for interesting psycho-pedagogical works and reflections starting from primary school (Di Sia, 2015).

It is possible to design multidisciplinary educational paths on the concept of infinity, through problematic and dialogic approaches. This allows us to evaluate the difficulties that may arise, the relational aspects, the aspects related to learning mathematics, the short- and long-term effects of the educational experience.

It is possible to study mathematics in different ways and the analysis of the historical evolution of concepts makes the study of definitions and theorems more comprehensible and easier. This statistically leads to an improvement in the students' profit levels. The need to explain to others the learned contents and the activity of deepening them in a non-traditional way allow a much greater internalisation of the proposed topics.

AI can help this type of process in various ways, but it does not seem to provide higher information compared to a serious, motivated, well-structured and in-depth traditional approach.

AI has an interesting potential to globally impact education, including the educational field. There are many possible uses of AI within the pedagogical-educational context, also considering the growing use of software and technological devices within the social and economic context.

However, particular attention should be paid to the possible ethical developments of AI regarding the protection of privacy of human beings in a world that is increasingly devaluing the human being in favour of political and economic aspects. The scientific community also questions the related ethical problems, the issues related to data security, the guarantee of true privacy, troubles arising also by an inadequate use of technological devices.

Although there are valid supports for developing teaching methodologies and practices, these technological devices remain today the prerogative of a small group of people who are part of the most developed society, who however do not always possess the necessary skills to be able to adequately apply them. This is also attributable, in the educational sector, to the lack of training of teachers with respect to software and digital devices.

Another important aspect on which people must reflect upon concerns the possible future acquisition by AI of a characteristic that it has not been able to replicate to date, namely the element of emotionality and consciousness, as this would lead to equating AI with human intelligence. However, it seems that it will never be able to do so, since consciousness is beyond matter, beyond this space and time. No algorithm can replicate the knowledge of meaning, the perception of truth, the recognition of beauty; it is a line that no machine will ever be able to cross (Faggin, 2024; Di Sia, 2022).

The scientific community and people must constantly reflect on the role that AI is assuming and will continue to assume within the society that we all are collaborating to develop day after day.

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